Jet and Rocket Propulsion

AE4451

LECTURE 4

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Overview

what we saw last time:

- examples applying the Reynolds Transport Theorem
  - stationary jet on test stand
  - stationary rocket on test stand
  - turbine system (~~partially~~)

today:

- thermodynamics continued: important concepts and application

Definitions

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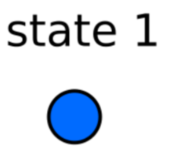
Property characteristic of a system, quantitatively evaluated

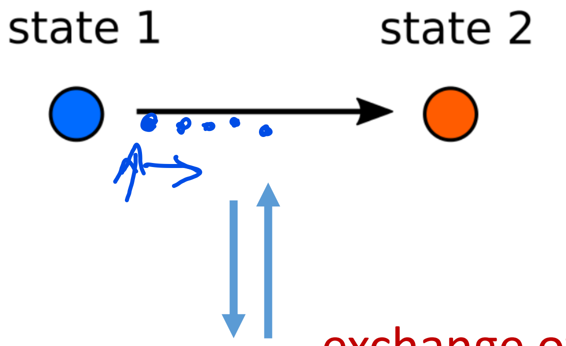
Thermodynamic state



condition of system (list of properties)

Equilibrium properties have unchanging values unless external conditions change

Thermodynamic process



- progressive evolution, from equilibrium to equilibrium
- with reversible process: can return to initial conditions

exchange of heat/work
with surroundings

Notions of system energy

Total energy = internal + potential + chemical + kinetic

$$(E_0, e_0) \qquad (E, e)$$

$$e_0 = e + \frac{1}{2}u^2$$ stagnation energy

internal energy only a function of the state, i.e. $e = e(p, T)$

Stagnation state

- state achieved if the gas is brought to rest via a fully reversible and adiabatic process without performing any work
- all of the energy converted to internal energy

Equations of state

- for a pure substance: 2 properties needed to define state
  other properties calculated from equation of state

Ideal gas relation (thermal state equation)

$$p = \rho RT$$

$$pV = RT$$

R : specific gas constant

$$p = \rho \frac{\overline{R}}{\overline{M}} T$$

$\overline{R}$: universal gas constant = 8.3143 J/mol.K

$\overline{M}$: molecular mas

Gibbs equation

$$ds = \frac{de}{T} + \frac{p}{T} dv$$

Caloric equations of state

$$h = h(T, \rho)$$

$$e = e(T, \rho)$$

Equations of state

Energy state equations (specific heats)

$$c_v = \frac{de}{dT} \qquad c_p = \frac{dh}{dT} \qquad c_p - c_v = R$$: specific gas constant

$$\gamma = \frac{c_p}{c_v}$$: adiabatic index, isentropic expansion factor, depends on atomic and molecular degrees of freedom

monoatomic $$c_p = \frac{5R}{2} \qquad c_v = \frac{3R}{2} \qquad \gamma = \frac{5}{3}$$

diatomic $$c_p = \frac{7R}{2} \qquad c_v = \frac{5R}{2} \qquad \gamma = \frac{7}{5}$$

Equations of state

Energy state equations (specific heats)

$$c_v = \frac{de}{dT} \qquad c_p = \frac{dh}{dT} \qquad c_p - c_v = R$$: specific gas constant

$$\gamma = \frac{c_p}{c_v}$$

now rewriting the Gibbs equation $$ds = \frac{de}{T} + \frac{p}{T}dv$$

$$\begin{aligned} Tds &= de + pdv \\ &= de + \underbrace{pdv + (vdp}\, - vdp) \\ &= de + d(pv) \quad - vdp \\ &= dh - vdp \end{aligned}$$

TdS

recall $$h = e + \frac{p}{\rho}$$

Equations of state

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we can now write the Gibbs equation as

$\frac{dh}{dT} = c_p$

$$ds = \frac{dh}{T} - \frac{v}{T}dp$$

$$dh = c_p dT$$
$$pv = RT$$

thus

$$ds = \frac{c_p(T)dT}{T} - \frac{R}{p}dp$$

what is the entropy change from one state (s1) to another (s2)?

$$\int_{s_1}^{s_2} ds = s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_p(T)dT}{T} - R\int_{p_1}^{p_2}\frac{dp}{p} = \int_{T_1}^{T_2} \frac{c_p(T)dT}{T} - R\ln\frac{p_2}{p_1}$$

$$s_2 - s_1 = [\phi(T_2) - \phi(T_1)] - R\ln(p_2/p_1)$$

same function

p-T-s state equation

Equations of state

p-T-s state equation

$$\underbrace{s_2 - s_1}_{\Delta s_{12}} = \left[\phi(T_2) - \phi(T_1)\right] - R\ln\left(p_2 / p_1\right) \quad = \int_{T_1}^{T_2} \frac{c_p(T)\,dT}{T} - R\ln\frac{p_2}{p_1}$$

for a calorifically perfect gas

$C_p \neq C_p(T)$

$C_p =$ constant

$C_v =$ constant

$$s_1 - s_2 = c_p \ln\frac{T_2}{T_1} - \underbrace{R\ln\frac{p_2}{p_1}}_{\text{unchanged}}$$

Processes: isentropic

from second law of thermodynamics, entropy can be written as

$$dS = \left(\frac{dQ}{T}\right)_{rev}$$

i.e., entropy change for reversible heat exchange at T

$$\left.\frac{dQ}{dt}\right|_{rev}$$

irreversible processes examples:

- friction is an example of an irreversible process
- heat/mass transfer with finite gradient

more generally $dS \geq \left(\frac{dQ}{T}\right)_{rev}$

$$ds = \frac{dQ}{dt}$$

Equations of state

p-T-s state equation

$$\underbrace{s_1 - s_2}_{\Delta s_{12}} = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

constant

temperature change from $T_1 - T_2$

general form

$$\frac{p_2}{p_1} = e^{(\Delta \phi_{12} - \Delta s_{12})/R}$$

entropy

for a calorifically perfect gas

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{c_p/R} e^{-\Delta s_{12}/R}$$

if isentropic process, then

temperature change

$$\frac{p_2}{p_1} = e^{\Delta \phi_{12}/R}$$

for a calorifically perfect gas

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{c_p/R}$$

$$= \frac{\gamma}{\gamma - 1}$$

Processes: adiabatic

as we saw previously

$$dE_0 = \delta Q_{in} - \delta W_{out}$$

work done by system

change in system energy

heat transferred to system

$$dE_0 = \delta Q - \delta W$$

for an adiabatic process (Q = 0) with reversible pV work: $dE_0 = -\delta W$

pV

$$\delta W = pdV = d(pV)$$

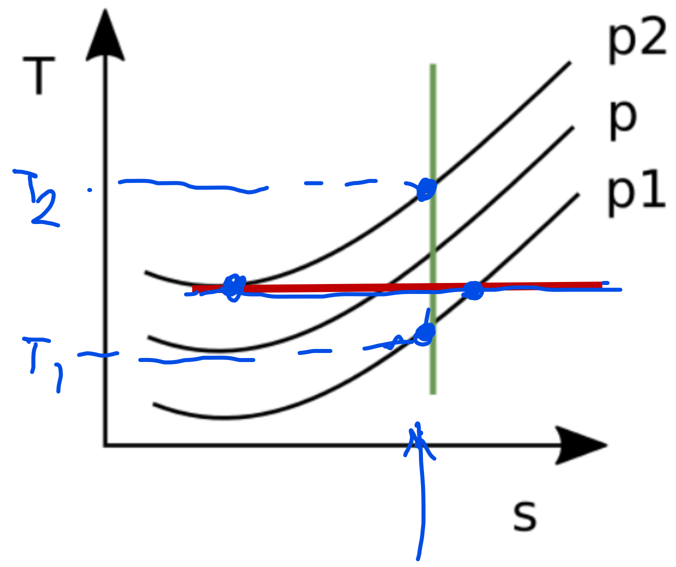
$$dE_0 = -dpV$$

$$d(\underbrace{E_0 + pV}_{\text{stagnation enthalpy}}) = 0$$

i.e. stagnation enthalpy does not change in such a process

p-T-s representation

to understand processes, we can use state diagrams



$$s_1 - s_2 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

higher entropy state – pressure decrease

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1} \right)^{c_p/R}$$

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1} \right)^{c_p/R} e^{-\Delta s_{12}/R}$$

isentropic assumption
higher pressure – increase temperature

Adiabatic efficiency

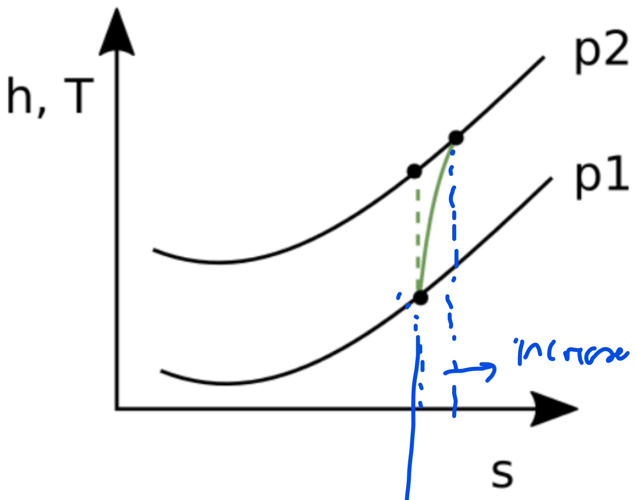
correct η

$$\eta = \frac{\Delta \text{ enthalpy across real device}}{\Delta \text{ enthalpy across idealized device*}}$$

higher

lower enthalpy

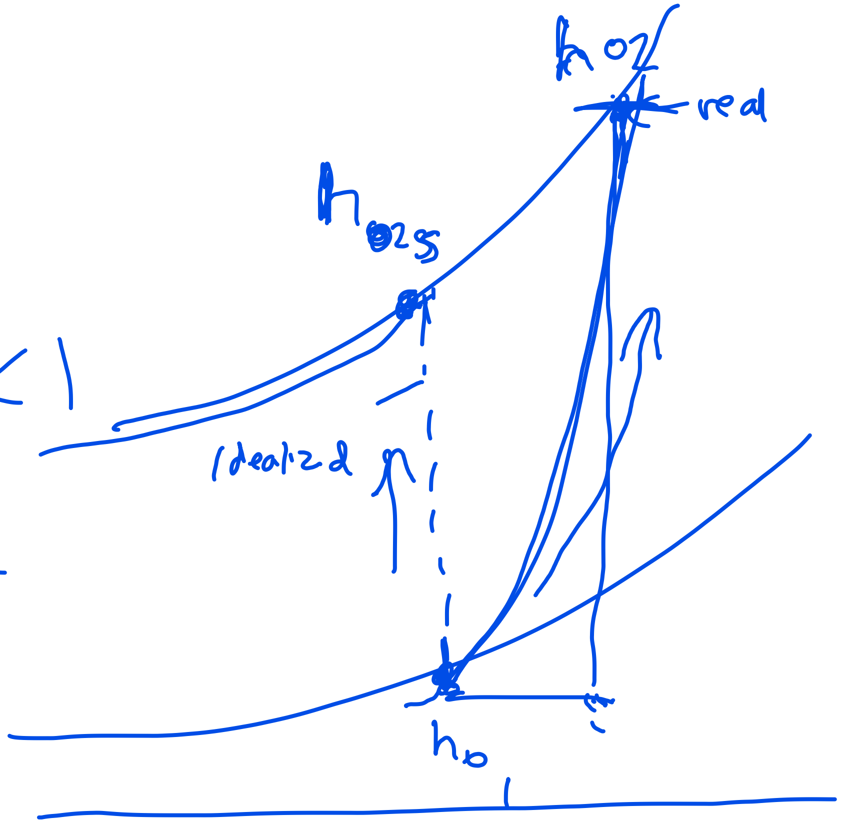
* reversible
adiabatic
uniform inlet/outlet pressures



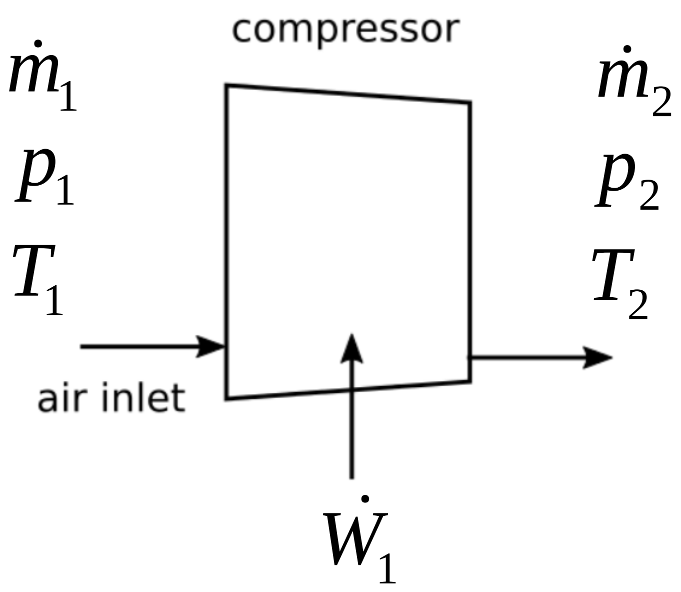
for compressor:

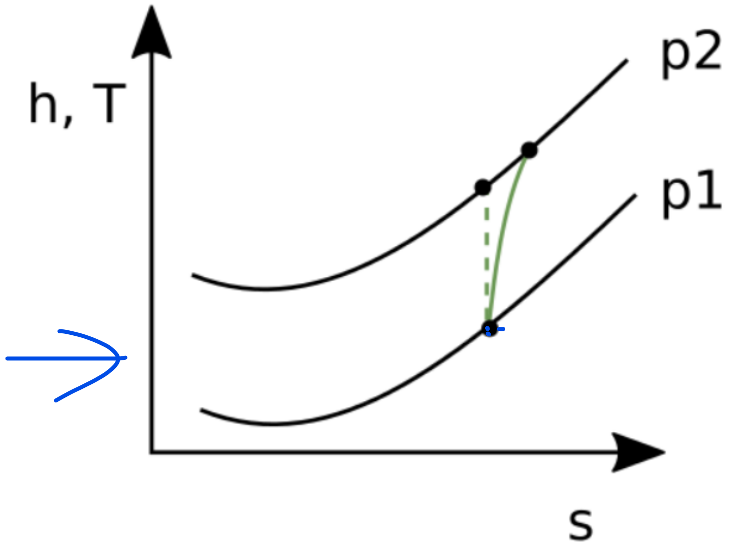
$$\eta_c = \frac{h_{o_{2s}} - h_{o_1}}{h_{o_2} - h_{o_1}} < 1$$

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1}$$



Example





$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1}$$

write:

- mass conservation
- energy conservation
- state equation relating temperature to pressure
  (hint: use the adiabatic efficiency)